

Department of the Navy
Bureau of Ordnance
Contract NOrd-16200
Task 5

AN EXPERIMENTAL
INVESTIGATION OF THE COLLAPSE
OF TRANSIENT CAVITIES IN LIQUIDS

by
Ieuan R. Jones

Hydrodynamics Laboratory
CALIFORNIA INSTITUTE OF TECHNOLOGY
Pasadena, California

Department of the Navy
Bureau of Ordnance
Contract NOrd-16200
Task 5

AN EXPERIMENTAL INVESTIGATION OF THE COLLAPSE
OF TRANSIENT CAVITIES IN LIQUIDS

Ieuan R. Jones

Reproduction in whole or in part is permitted for any
purpose of the United States Government

Hydrodynamics Laboratory
California Institute of Technology
Pasadena, California

ABSTRACT

In the first section of this report a review of some of the attempts to measure cavity collapse pressures is presented.

The second section deals with the experimental measurement of the stresses developed in a solid subjected to cavitation. Single, hemispherical cavities were created in water at the end of a pressure bar. Both the generation and collapse of such cavities gave rise to transient stress pulses which traveled along the bar and were detected by means of an X-cut quartz crystal. The charge appearing on the opposite faces of the quartz disc was allowed to develop a voltage which was subsequently amplified and recorded photographically from an oscilloscope. The oscillograms obtained in this manner yielded information regarding both the total lifetime of the cavity and the peak amplitude of the force developed during its collapse.

Concurrent with the recording of the stress pulses, high speed photographs covering the total lifetime of the cavity were taken. These photographs furnished information regarding both the maximum diameter attained by the cavity and the time history of collapse of the cavity.

A preliminary report of this research was presented at the Fifty-Eighth Meeting of the Acoustical Society of America, Cleveland (Oct. 1959).

I. INTRODUCTION

The term, cavitation, is given to a group of phenomena of great importance in both marine and hydraulic engineering in which cavities filled with vapor or vapor and gas are formed in the interior of a liquid. These cavities may be formed in various ways; in general, if, at some point in a liquid, the existing fluid pressure equals the vapor pressure at the particular temperature, then the liquid will vaporize and cavities will form. If the fluid pressure subsequently fluctuates above and below the vapor pressure, there will be an alternate collapse and formation of the cavities. Cavitation is inherently an unstable phenomenon.

The possibility of cavitation in hydraulic machines was referred to by Euler^{1*} in a memoir dealing with the theory of turbines. However, the earliest laboratory experiments were described by Reynolds² who observed cavitational phenomena in water flowing through a tube with a local constriction.

At the turn of the century, a peculiar destruction of the screw propellers of fast steam boats was observed by many marine architects. It was noticed that these particularly severe erosive destructions appeared after only a few hours running and were enough to render the screw unfit for work. These newly observed destructions were not confined to screw propellers alone. A similar type of damage was concurrently noted in hydraulic machinery, the damage being limited to those areas of the components which either moved in water or came into direct contact with a water current.

The detrimental effects of cavitation in both marine and hydraulic

* Superscripts refer to reference numbers in bibliography

engineering stimulated interest in the mechanics of cavitation with the object of clarifying such questions as the origin, growth and collapse of the cavities. In particular, there has been during the last 50 years an impressive accumulation of data and ideas concerning the specific topic of cavitation damage. Many theories based on, for example, mechanical, mechanochemical, electrochemical and thermochemical concepts have been postulated in an attempt to explain the mechanism of cavitation damage. Of these, the concept of mechanical damage has undoubtedly been the most popular one and has been well established by theory and experiment. The possibility of chemical action has not been neglected by exponents of this theory; they have merely considered it unlikely to be of any particular significance in cavitation damage. According to the purely mechanical theory of damage, the high pressures associated with the collapse of transient cavities on or very near to a surface produce plastic deformation (cold-working) and eventual fracture of brittle materials or fatigue failure of ductile materials.

On the theoretical side, the problem of the collapse of a single cavity has been attacked by many workers. The first important work was carried out by Lord Rayleigh³ who obtained a solution for the collapse of a spherical cavity in an infinite, incompressible, and nonviscous fluid medium. It was assumed that the pressure within the cavity was zero or constant during the collapse process. This solution, however, gave infinite values for cavity wall velocity and pressure at the collapse point and it was apparent that the problem of cavitation bubble collapse in a real liquid required a far more rigorous treatment than that developed in the simple Rayleigh theory; one that would take into account diffusion and vaporization processes, compressibility and energy

dissipation. Nevertheless, this solution, together with the approximate solutions which followed later, gave an adequate picture of the hydrodynamics of the motion as long as the cavity radius was large compared with the minimum radius.

The assumption of an incompressible liquid, while satisfactory for slow phases of the motion, was clearly untenable during the last stages of collapse where high radial velocities are encountered. One of the most significant and comprehensive theoretical treatments of the subject of cavity collapse is that by Gilmore⁴ who generalized the analysis to include higher order compressibility terms as well as the effects of viscosity and surface tension. It is interesting to note that the inclusion of these refinements to the case of the collapse of a cavity having a constant (or zero) internal pressure does not prevent the bubble wall velocity from increasing monotonically as the radius decreases and becoming infinite at the point of collapse. It is, of course, immediately apparent that infinite pressure at collapse can be eliminated by assuming that the pressure in the cavity increases with decreasing bubble radius. Such a pressure rise can occur if either the cavity contains some permanent gas or if, at some point during the collapse, the vapor may not be able to condense as rapidly as is required by bubble wall motion and consequently begins to behave as a permanent gas.

In view of the experimental nature of the present investigation, it would seem appropriate at this point to review some of the attempts to measure cavity collapse pressures that have been made during the last 40 years.

The experimental investigations began with the work of Parsons who held the view that cavitation damage resulted from a water hammer

action arising from the collapse of vacuous cavities at a surface. The cone experiment devised by Parsons⁵ demonstrated that it was possible to punch a hole through a metal disc by concentrating thereon the energy of the momentum of a large volume of water moving down the cone. These results were cited as proof that very high local pressures could be developed at the site of collapse of vacuous cavities.

In the decade between 1926 and 1936 many experiments were carried out to examine the damage suffered by various materials when subjected to the destructive action of cavities which arise when water flows through a pipe containing a local constriction. Such an experiment had been attempted at an earlier date, but without much success, by Parsons and Cook⁶. Föttinger⁷, Schrotter^{8,9,10}, de Haller¹¹ and Mousson¹² obtained desired results by using pipes in which the cross sectional area at the narrowest part of the pipe was a few square centimeters and in which the velocity of the water reached 100 meters/sec. By placing a specimen of the material under investigation in the path of the movement of the cavities, these authors obtained intensive destruction of a series of metals and alloys.

P. de Haller attempted a direct measurement of the collapse pressures. Instead of inserting a test specimen into the venturi, he placed a piezoelectric pressure detector and displayed the voltage generated across the quartz crystal on an oscilloscope. The records obtained by de Haller showed the presence of impulse pressures of the order of 300 atmospheres. It should, however, be mentioned that the transient peak pressures could not have been fully resolved because the natural vibration of the crystal as well as the frequency response of the oscilloscope were of the same order of magnitude as the duration

of the pressure impulses ($\sim 10\mu s$).

In 1932, Gaines¹³ developed a new method of obtaining cavitation destruction. He excited intense longitudinal vibrations in a nickel rod by making use of the magnetostriction effect. When the vibrating rod was immersed in water, cavities appeared on the end surface of the rod and the surface was intensely damaged. These effects were caused by the alternating pressure field which exists near the vibrating end of the rod. During the part of the cycle when the pressure falls to the value corresponding to the vapor pressure, cavities were formed which collapsed and disappeared during the remaining part of the pressure cycle. Gaines' method was improved and used by a number of investigators, among them being Hunsaker¹⁴, Kerr¹⁵, Schumb, Peters and Milligan¹⁶, Beeching¹⁷, Nowotny¹⁸ and Poulter¹⁹.

The object of the experimental investigations of nearly all the authors mentioned so far, was to find materials which could withstand sufficiently the destructive action of cavitation. Since most of the materials had static yield strengths of 10^5 - 10^6 lb/in², it was at first concluded that peak collapse pressures were in this range.

Concurrent with this work on damage to materials was a line of experiments carried out by Cook²⁰, Schwarz and Mantel²¹, Ackeret and deHaller²² and others which showed that the mechanical damage produced by small water drops closely resembled cavitation damage and that the drops need not be traveling at exceptionally high speeds to cause damage as long as there were a sufficient number of impacts. Impact pressures of 10^3 - 10^4 lb/in² were sufficient to cause pitting of the specimen and accordingly it was concluded that peak pressures in collapsing cavities were of this smaller order of magnitude. There

is implicit in these experiments the Parsons-Cook assumption that cavitation erosion is due to a water hammer collapse of cavitation voids. This assumption has been justified to a certain extent by the work of Kornfeld and Suvorov²³ who, as a result of experimental work carried out employing a magnetostrictive method of generating cavitation, ascribed the destructive action of cavities produced in this manner to the direct blows of water against the surface exposed to cavitation. These water blows are assumed to arise as a direct consequence of an asymmetric collapse of unstable surface cavities.

During the past 15 years, single transient cavities have been created and studied by a number of investigators. Osborne²⁴ employed an experimental technique whereby small air bubbles were introduced into the liquid in order to assist the formation of the cavity. Cavities were thus obtained whose air content was known. Pressure variations in the water arising from the collapse of a cavity were recorded by means of a small quartz pressure gauge which could be placed at a variable distance from the cavity. The voltage developed across the quartz crystal was displayed on an oscilloscope. Osborne observed that his oscillograms indicated multiple pressure peaks, the interval between the peaks depending on the initial size of the cavity. He also noted that the amplitude of the initial pressure peak depended on the maximum radius of the cavity and was especially dependent on the air content of the cavity, the amplitude decreasing with increased air content. An interesting point arose in the discussion following this paper. B. G. Rightmire expressed the opinion that the measured pressure was only remotely connected with that occurring in the region of collapse, since the energy liberated at collapse was quickly trans-

formed into energy of elastic waves in the water. He reported measurements which he had carried out showing an essentially discontinuous rise in pressure at collapse, the high pressure being of very short duration of the order of $10\mu\text{s}$ or less. These transient high pressures were held to be responsible for the damage done by cavitation and, as such, Rightmire felt that attention should be focused on them.

A remarkable step forward was achieved in 1948 by Knapp and Hollander²⁵ who, in an excellent series of photographs, showed several cycles in the oscillations of cavities in a flowing liquid. Meuller²⁶ had previously published photographs showing cavitation on a hydrofoil but apparently had not observed cavity oscillation. By comparing the experimental results with Plesset's²⁷ equation of motion of the cavity wall integrated in this instance for the pressure field corresponding to the conditions of the experiments of Knapp and Hollander, it was shown that the collapse of cavitation bubbles agreed to quite a degree with Rayleigh's analysis. This justified the assumption that mechanical forces are the main factor governing the motion during the greater part of the collapse period. As a typical example, Knapp and Hollander estimated from their photographs that the final collapse velocity of a 0.14 in. radius bubble was 765 ft/sec which on substitution into the water hammer equation ($P = \rho c V$ where P is the pressure developed, ρ is the density of the liquid, c is the velocity of sound in the liquid and V is the velocity of the liquid at impact) yielded a collapse pressure of 50,000 lb/in².

Chesterman²⁸ examined the dynamics of single transient cavities formed in water and certain organic liquids. These cavities were generated by rapidly decelerating a solid-liquid interface and the

pressure changes near the cavities were recorded by means of a Rochelle salt pressure gauge situated in the liquid. Chesterman showed that at each volume minimum of the cavity the pressure rises in the liquid to a sharp maximum, the peak pressure being reached in the order of $5-10\mu\text{s}$. Examination of cavities formed in liquid mixtures having different surface tensions did not lead to conclusive results.

Harrison²⁹ studied single cavities produced by two experimental techniques. In the first instance the cavities were produced in a venturi nozzle and in the second they were generated by the exploding wire method. The pressure variations in the water were measured at a distance of 10 cms from the cavity by means of a tourmaline pressure gauge. He confirmed that the noise in single bubble cavitation consists principally of a transient pressure pulse ($\sim 15\mu\text{s}$ duration) associated with the collapse of the bubble. By assuming that the minimum radius of the cavity was 0.025 cm and that the pressure varies inversely with distance, Harrison estimated, as an order of magnitude, that the maximum pressure at the bubble was 4000 atmospheres.

Mellen³⁰ employed both the exploding wire and spark gap techniques for generating single cavities and he determined the pressure field produced by the collapse of the bubble by means of a barium titanate pressure gauge situated at a distance of 50 cm from the bubble. In the interval corresponding to subsonic flow, the pressure was found to increase as the bubble collapsed. The pressure then suddenly jumped to a higher value and rapidly decayed to zero. The author assumed this rapid increase to a higher value to be due to a shock wave. Using Gilmore's theory for the collapse of the cavity and finite amplitude wave theory for the pressure wave, Mellen found that the value of the

pressure amplitude characteristic at the shock front corresponded to a cavity wall velocity approximately equal to the velocity of sound. Since the measured peak pressure indicated that the radial velocity of the cavity wall was very nearly equal to the velocity of sound and since the peak pressure amplitude had been strongly attenuated in the water, Mellen concluded that the peak radial velocity must have been in excess of the velocity of sound.

Glikman, Tekht and Zobachev³¹ using a magnetostriction oscillator and Plesset and Ellis³² using a barium titanate transducer to generate standing pressure waves in the liquid have shown that the failure produced in a cavitated region has precisely the character of fatigue failure. In both investigations, the changes in metal structure at various depths were carefully examined by means of photomicrographs and X-ray diffraction patterns. Plesset and Ellis showed that deformations due to cavitation impacts appear almost immediately with metals having ultimate tensile strengths of the order of $50,000 \text{ lb/in}^2$, but that the onset of damage is very slow with tungsten or titanium 150-A alloy (tensile strength $130,000 \text{ lb/in}^2$). Such result indicated that stresses due to cavitation must have a lower limit of the order of $50,000 \text{ lb/in}^2$. Experiments in which cavitation damage was produced in a chemically inert environment led the authors to conclude that chemical effects are not of primary importance.

Direct photoelastic evidence of the existence in solids of strain waves originated by bubble collapse was provided by the work of Ellis³³ and Sutton³⁴. Using an ultra high speed motion picture camera, ultrasonic cavitation bubbles were photographed collapsing on the surface of a photoelastic specimen and photographs of the resulting transient

isochromatic patterns in the specimen due to strains caused by cavitation were also obtained. The dynamic properties of the photoelastic material were obtained in order to permit a quantitative interpretation of the phenomenon. The results indicated that the stresses due to cavitation were of the order of 2×10^5 lb/in². A photocell, used to detect the transient strains, indicated that the time duration of the strains was about 2 μ s.

The application of photoelastic techniques to the cavitation problem has the advantage over the technique of using piezoelectric gauges to detect the collapse pressure pulses in that the former shows the precise point of maximum pressure rather than the integrated pressure over a relatively large area. Moreover, the calibrations necessary in the experiment are those relating to the properties of the photoelastic material and they do not involve electronic apparatus. It should be mentioned, however, that the interpretation of 3-dimensional transient strain patterns necessitates, in general, an assumption of symmetry and, furthermore, the author states that a fuller study of the dynamic behavior of photoelastic solids at higher frequencies and higher stress levels would have to be undertaken before accurate direct measurement of cavitation stresses could be made. It is interesting to note that Ellis suggests that an application of the piezoelectric technique to the measurement of the high stresses arising at cavity collapse would be useful in providing a correlation with the photoelastic method. Such an approach has been used by Jones and Edwards³⁵; this work will be discussed in the following section.

Evidence for the existence of a radiated shock wave originating at the collapse of a cavity may be found in the experimental observation

of, among others, Osborne, Chesterman, Harrison and Mellen. Further, Güth³⁶ and Jones and Edwards have published Töepler-Schlieren photographs showing shock waves being radiated in liquid at the collapse of transient cavities.

From the foregoing survey of both theoretical and experimental cavitation studies it is fair to conclude that, at the instant of collapse of a cavity in a liquid, a very high pressure is developed at the site of collapse and a pressure wave is radiated into the liquid. If the cavitation bubble collapses directly onto the surface of a solid, high local stresses can arise at the surface. The origin of surface damage can be considered to lie in these high collapse pressures whether the actual removal of material from the surface be assumed to be purely a result of mechanical fatigue or to be due to an association of chemical, electrochemical, thermoelectric and mechanical effects. It is also evident from this survey that very little work has been carried out in an attempt to measure the pressure pulse developed at the actual site of collapse of a transient cavity. In a recent critique, Eisenberg³⁷, discussing the dynamics of transient cavities, defines one of the basic problems of cavitation thus: "In addition to the photographic observations at high speed, there remains a need for the measurement of the magnitude and character of the pressure pulse in the last period (of collapse)." The present investigation was initiated with the object of examining this pressure pulse since it appears that even an approximate measurement of collapse pressures would represent a worthwhile contribution to progress in cavitation research.

II. THE EXPERIMENTAL MEASUREMENT OF THE STRESSES DEVELOPED IN A SOLID SUBJECTED TO CAVITATION

As was mentioned in the last section, the measurement of the high pressure developed at the site of collapse of a transient cavity is a subject which has received scant attention and the results reported in the literature are fragmentary and inconclusive. This has been largely due to the difficulties encountered in designing a gauge capable of recording large amplitude pressure changes which take place in times of the order of $\sim 10\mu\text{s}$ or less. Ordinary mechanical gauges which can withstand the high pressures are of little use owing to their completely inadequate high frequency response. A large proportion of the more successful pressure measuring devices which have been designed for the recording of rapidly varying pressures employ as their sensitive element some crystalline material which exhibits the piezoelectric effect. However, the direct use of these gauges is limited to be measurement of comparatively low pressures on account of the fragility of the crystal elements.

In recent years, physicists and engineers have become increasingly aware of the potentialities of the pressure bar which, in the original form devised by Hopkinson³⁸, could be used satisfactorily to measure maximum pressures and, with more difficulty, to measure the time during which the pressure exceeded any given value. Hopkinson's pressure bar, however, could not furnish a continuous relationship between the measured pressure and time.

Some of the disadvantages of the original pressure bar method can be avoided by using an electrical version of the Hopkinson bar. Such a development of the pressure bar technique was first described

by Davies³⁹; it permitted much smaller pressures to be measured and made it possible to determine the complete pressure-time relationship. The application of a pressure to one end (the 'pressure end') of a cylindrical bar gives rise to a stress pulse which travels along the bar. The radial displacement, or the longitudinal displacement, at the distant end of the bar (the 'measuring end') is used to produce a change in the capacity of a suitable condenser unit. This change of capacity is converted into a voltage pulse which is displayed on a cathode ray oscilloscope and recorded photographically. From the oscillogram, which gives the variation with time of the measured displacement of the bar, the variation of the applied pressure with time can be deduced. Another method of measuring the stress wave pulse in the bar is to record the resultant strain at the surface of the bar by means of a resistance strain gauge (Thomas⁴⁰) or, alternatively, by means of a piezoelectric strain gauge consisting of a small wafer of barium titanate (Ripperger⁴¹). Edwards⁴² has described a piezoelectric method of measuring the stress variations over the cross-section of a duralumin pressure bar by means of an X-cut quartz disk. The disk is sandwiched between two cylindrical bars of the same diameter and the composite bar so formed was shown to behave as a continuous bar for longitudinal stress waves provided that the value of the acoustic impedance of the duralumin closely matched that of the quartz. The polarization produced in the quartz at any instant is proportional to the average stress throughout the body of the crystal; the charges appearing on the opposite plane faces of the quartz disk are allowed to develop voltage signals which are subsequently amplified and displayed on a cathode ray oscilloscope. The advantage of using the piezoelectric element in this manner is that

the uncertainty caused by oscillations in the gauge output due to the excitation of the natural frequencies of the element is reduced without undue sacrifice of the high frequency response of the element.

Recently, Jones and Edwards have studied the cavity collapse problem by means of the piezoelectric pressure bar technique. Single, transient hemispherical cavities were created in water at the pressure end of a composite duralumin - flint glass - lead pressure bar employing a circular disk of X-cut quartz as its sensitive element. At the collapse of a cavity a transient stress pulse was propagated along the bar which was detected by the quartz crystal. The oscillograms obtained in this experiment furnished information regarding the peak force developed at the collapse of the transient cavity. In particular it was possible to plot the relationship between the total lifetime of a cavity and the peak amplitude of the force developed at the collapse of that cavity. Some information on the character of the collapse pulse was obtained from fast sweep experiments but unfortunately the effective writing speed of the recording system was such that much was left to be desired in the quality of the experimental oscillograms obtained during that part of the investigation. No high speed framing photographic observation of the collapse of the cavities was carried out during the course of this work.

Used by itself, the piezoelectric pressure bar will only produce information regarding the peak force developed during collapse. Thus, Jones and Edwards could only estimate a lower limit of 10,000 atmospheres for the peak pressure. This value was based on the assumption that the only force exerted on the pressure end of the bar was that due to the contents of the cavity and on the observation that the cavities

collapsed to a diameter which was at least that of the tungsten needle used to produce the cavity generating spark. It was felt that an ultra high speed photographic technique employed in conjunction with this piezoelectric pressure bar technique would go far towards reaching a better understanding of the collapse process. An ultra high speed camera which utilized a Kerr cell shutter was available for the present investigation; its previous application to cavitation problems has been described by Ellis³³. Recording equipment having a good overall writing speed was also available for the project. Thus at the start of this experimental project it was envisaged that we would be primarily concerned with:

- (i) obtaining high speed photographs covering the total lifetime of a cavity. These pictures would show the manner in which the cavity behaved during its time history and would furnish information regarding maximum diameter etc., attained by the cavity. A pressure bar oscillogram would be taken simultaneously with the above.
- (ii) displaying the collapse pulse on a fast sweep ($\sim 2\mu\text{s}/\text{cm}$) and obtaining good quality oscillograms. Concurrent with this, ultra high speed pictures of the collapsing cavity would be taken. By doing this, it was hoped that some knowledge of the (pressure-time) relationship existing within the cavity during the last $20\mu\text{s}$ of collapse would be obtained.

Phase (i) of the experimental program has been completed and the present report deals with this part of the project.

III. DESCRIPTION OF APPARATUS

A schematic diagram and a photograph of the experimental set-up are shown in figures 1 and 2.

Pressure bars, tank and cavity generation.

Cylindrical duralumin - flint glass - lead pressure bars of the type described by Jones and Edwards were employed to measure the forces developed by the collapsing cavities. One end of the bar passed through the bottom of a 12"x12"x12", 1/2" wall, Lucite tank via an oil seal. The tank contained water which had been allowed to stand in the tank for some time and which was assumed to be saturated with air at room temperature and pressure.

Hemispherical cavities were created on the end of the bar which protruded into the tank by discharging a charged condenser through a gap formed between a tungsten needle and the end of the bar. A Type 5C22 thyratron was used as the switching element in the discharge circuit; a total discharge time of about $10\mu\text{s}$ was obtained. A device whereby the length of the spark gap could be varied was fixed to the lid of the tank. Various sizes of cavities were produced by varying the spark gap length and the charging potential and capacity of the condenser. Transient stress pulses are propagated in the bar during both the initial growth and the final collapse periods of the cavity lifetime and are detected by an X-cut quartz crystal which is sandwiched between the flint glass and lead sections of the composite bar and which measures the average stress over the cross-section of the bar. Since the X-cut quartz crystal is a high impedance detector, difficulty was encountered with stray electrostatic pick-up and consequently the bars were mounted

III. DESCRIPTION OF APPARATUS

A schematic diagram and a photograph of the experimental set-up are shown in figures 1 and 2.

Pressure bars, tank and cavity generation.

Cylindrical duralumin - flint glass - lead pressure bars of the type described by Jones and Edwards were employed to measure the forces developed by the collapsing cavities. One end of the bar passed through the bottom of a 12"x12"x12", 1/2" wall, Lucite tank via an oil seal. The tank contained water which had been allowed to stand in the tank for some time and which was assumed to be saturated with air at room temperature and pressure.

Hemispherical cavities were created on the end of the bar which protruded into the tank by discharging a charged condenser through a gap formed between a tungsten needle and the end of the bar. A Type 5C22 thyatron was used as the switching element in the discharge circuit; a total discharge time of about $10\mu s$ was obtained. A device whereby the length of the spark gap could be varied was fixed to the lid of the tank. Various sizes of cavities were produced by varying the spark gap length and the charging potential and capacity of the condenser. Transient stress pulses are propagated in the bar during both the initial growth and the final collapse periods of the cavity lifetime and are detected by an X-cut quartz crystal which is sandwiched between the flint glass and lead sections of the composite bar and which measures the average stress over the cross-section of the bar. Since the X-cut quartz crystal is a high impedance detector, difficulty was encountered with stray electrostatic pick-up and consequently the bars were mounted

in brass screening housings, which could be attached to the bottom of the tank. Oil seals were used to support the bars in the housings.

Figure 3 shows a photograph of a 1/2" diameter pressure bar lying in its supporting tube; the screening housing is also shown.

Electronic recording apparatus.

The charge appearing on the two plane faces of the quartz disk when it is traversed by a stress pulse was allowed to develop a voltage across a capacitance (approximately 1000 μ uf) connected in parallel with the disk. This voltage was amplified using a Tektronix Type L plug-in unit and was subsequently displayed on one channel of a Tektronix Type 551 oscilloscope, the second channel being used to provide the oscillogram with a datum line. The traces were photographed with a Polaroid Land camera using Type 46L film.

High speed photographic equipment.

High speed photographs covering the total lifetime of a cavity were obtained simultaneously with the recording of the stress pulse developed in the pressure bar at the collapse of that cavity. The photographic system used has been fully described by Ellis³³. It consists essentially of a Kerr cell shutter used in conjunction with a rotating mirror camera. Using this system, a sequence of pictures covering the lifetime of a cavity and taken at rates of up to 100,000 frames per second with exposure times of approximately 10^{-7} sec could be obtained.

The cavities were backlighted with a "steady" light source. A G. E. Type 524 flash lamp was used, the lamp being fed with a square pulse of current of about two milliseconds duration obtained from a lumped parameter delay line. There was a lapse of about 200 μ s before the light output reached its peak value.

IV. EXPERIMENTAL PROCEDURE AND RESULTS

The experiments were carried out using both a 1/2" and a 1/4" diameter pressure bar. The X-cut quartz crystals incorporated in the pressure bars were not calibrated in these experiments; calibrations carried out during the course of some previous work (Jones and Edwards) had indicated that there was hardly any difference between the experimentally measured value of the piezoelectric strain modulus and the value of $d_{11} = 2.21 \times 10^{-14}$ coulombs/gram quoted by Mason⁴³.

The experimental procedure was as follows. An appropriate picture framing rate was chosen and the repetition rate of the Kerr cell pulser unit was set accordingly. The turbine of the high speed camera was then allowed to reach a speed sufficiently high to prevent overlapping of consecutive frames. When this turbine speed was reached a delay multivibrator was triggered manually. This unit delivered a nondelayed triggering pulse to the lamp unit and approximately $200 \mu s$ later (by which time the light from the lamp had reached its maximum value) a pulse which triggered the underwater spark gap, the Kerr cell pulser and the oscilloscope. Thus a high speed photographic record covering the lifetime of the cavity and an oscillogram showing the output of the pressure bar were obtained simultaneously.

Pickup signals, obtained from a loop placed near the Kerr cell were superimposed on the oscillograms. These furnished the records with timing markers of an accurately known frequency, namely the repetition rate of the Kerr cell pulser. This procedure also allowed one to correlate the pressure bar reading and the high speed photographs.

Immediately after each experimental recording a calibration of the electronic equipment was carried out by feeding a sine wave of known amplitude into the input of the amplifier and subsequently photographing the output waveform displayed on the cathode ray tube. The results shown in figures 4 and 5 are typical of those obtained in these experiments. The high speed photographs are shown together with the oscillogram of the output of the pressure bar. Three pulses can readily be seen in the oscillograms. By correlating in time a set of photographs and the corresponding oscillogram it was found that the first pulse is produced at the formation of the cavity while the second pulse occurs during the last period of collapse of the cavity. After collapse the cavity rebounds and undergoes another growth-collapse cycle. The third and much smaller pulse seen in the oscillograms occurs at the collapse of this rebound cavity. The time T between the start of the formation pulse and the occurrence of the peak force in the first collapse pulse was taken to be the cavity lifetime.

The equation relating the charge Q liberated to the Force F applied to the major faces of the quartz crystal is

$$Q = d_{11} F. \quad (1)$$

Since this charge is allowed to develop a voltage V across a known capacitance C, equation (1) may be rewritten

$$C V = d_{11} F$$

or

$$F = \frac{C V}{d_{11}}. \quad (2)$$

If the value of V is known at a time t, then equation (2) enables the

value of the force F to be determined at that same time t . The value of V can be determined from the experimental oscillogram as follows. If δ_c is the vertical distance, measured on the oscillogram, between any two points of the calibrating sine wave (e. g. the distance between the crest and trough of the waveform) then to δ_c there corresponds a known voltage input V_1 to the amplifying system. It then follows that if δ is the vertical deflection of the experimental trace at a time t , the magnitude of the voltage V across the input terminals of the amplifier at time t is

$$V = \frac{\delta}{\delta_c} V_1$$

and hence the value of the force F at the same time t is

$$F = \frac{C}{d_{11}} \cdot \frac{\delta}{\delta_c} V_1.$$

In particular

$$F_m = \frac{C}{d_{11}} \cdot \frac{\delta_m}{\delta_c} V_1$$

where F_m is the peak force developed during the first collapse of the cavity and δ_m is the corresponding peak vertical deflection measured on the experimental record. The results obtained in these experiments are summarized in table 1.

V. DISCUSSION OF EXPERIMENTAL METHOD AND RESULTS

The pressure bar technique is subject to certain inherent limitations. In particular:

- (i) We assume that the stress pulse is propagated along the bar without distortion. This assumption is only valid when the wavelengths of the elastic waves concerned in the propagation of the pulse are large compared with the lateral dimensions of the bar. When this condition is satisfied, the waves, and hence the pulse, travel along the bar with a velocity $C_o = \sqrt{\frac{E}{\rho}}$ where E and ρ are respectively Young's Modulus and density of the material composing the bar.
- (ii) We assume that the pressure in the stress pulse is eventually uniform distributed over the cross-section of the bar even when the force acting on the pressure end is concentrated over a small area surrounding the center and is sufficiently large to produce local plastic deformation. This uniform distribution of pressure does occur provided that the duration of the pulse due to the applied pressure is not so short that dispersion is important and provided the length of the bar is more than 4 diameters.
- (iii) We assume that after the initial period during which the pressure becomes uniformly distributed across the cross-section of the bar, the waves propagated in the bar are elastic waves i. e. the stress at any point in the bar lies within the region where the (stress-strain) curve for the material of the bar is linear and reversible. The upper

limit to the stresses which can be measured with a particular bar is determined by the elastic properties of the material composing the bar.

It will be noted that no mention of spark gap length is made in the data presented in table 1. Although the gap length could be varied at will during the course of the experiments, the slight pitting of the bar surface which occurred with each discharge prevented an absolute measurement of this length being made. However, some qualitative observations could be made regarding cavity generation. It was found that for a nominally constant gap length, the maximum radius attained by a cavity increased with increasing electrical energy. Furthermore, it was found that the maximum radius increased with increasing spark gap length for a given amount of electrical energy. If the gap was made too large, however, no discharge occurred.

Figure 6 shows the plot of the (F_m, T) data obtained with the 1/2" and 1/4" diameter pressure bars. The corresponding data obtained by Jones and Edwards is plotted on the same graph for comparison. It is seen that there is good agreement between the two sets of results. An experimental (F_m, T) point obtained by Ellis⁴⁴ using the photoelastic technique mentioned in the introduction to this report is also included in figure 6. It is interesting to note that this result is consistent with the results obtained with the piezoelectric technique.

Jones and Edwards have reported that abrupt changes occur in the character of the (F_m, T) curves at values of T of approximately 460 s and 1100 s for the 1/4" and 1/2" diameter bars respectively. For values of T larger than these, the results were not repeatable. This anomalous behavior at large values of T was attributed to the

fact that in this region the maximum cavity diameter exceeded the pressure bar diameter and consequently the collapse of the cavity was irregular. This assumption was tentatively confirmed by rotating-drum camera streak photographs. Figure 7 shows the life-cycle of a cavity which attained a diameter larger than the diameter (1/2") of the pressure bar upon which it was formed. It is seen that this cavity does indeed collapse in a manner far different from that shown in figures 4 and 5. The peak force developed at the collapse of this cavity is smaller than that which would have resulted had the cavity collapsed in a uniform hemispherical way. It appears that instability in the collapse stage tends to reduce the damage potential of a cavity.

Figure 8 shows the experimentally determined relationship which exists between the maximum radius R_o attained by a cavity during its life-cycle and the total lifetime T of that cavity. For the range we have examined, the following linear relationship holds between R_o and T

$$T = k R_o$$

where $k = 1800$ when R_o is expressed in cms. and T in μ s. Now, $T = t_g + t_c$ where t_g = time taken by the cavity to grow to its maximum radius R_o , and t_c = time taken by the cavity to collapse from $R = R_o$ to $R = 0$. Therefore,

$$t_c + t_g = k R_o$$

i. e.

$$t_c = \frac{k}{\left[1 + \frac{t_g}{t_c}\right]} \cdot R_o$$

An analysis of the high speed photographs (table 2) revealed that t_g/t_c was constant in our range of interest and had a value of 0.94. Thus

the relationship between t_c and R_o is

$$t_c = \frac{1800}{1.94} \cdot R_o$$

i. e.

$$t_c = 928 R_o$$

where R_o is expressed in cms and t_c in μs .

Due to the spherical symmetry of Rayleigh's theory of collapse, this theory can be applied to the collapse of a hemispherical cavity having a zero or arbitrarily constant internal pressure on a semi-infinite body in an incompressible non-viscous liquid provided frictional losses at the liquid-solid interface are neglected. Rayleigh's theory yields the result

$$t_c = 0.915 R_o \sqrt{\frac{\rho}{P}}$$

where ρ = the density of the liquid in grams/cm³ and P , the external pressure in dynes/cm². On substituting the relevant values of ρ and P into this expression we obtain

$$t_c = 908 R_o$$

where R_o is expressed in cms and t_c in μs . Thus the theory predicts the value of t_c for a given R_o with a fair degree of accuracy. A further detailed analysis of the high speed photographs showed that the time history of collapse of the cavities followed closely that computed from Rayleigh's theory. This fact is illustrated in the graph shown in figure 9 where the experimental and theoretical time histories of collapse are plotted in a non-dimensional manner.

From the point of view of assessing the damage potential of various cavities, attention must be focused on the value of the peak pressure, as distinct from the peak force, developed at the seat of

collapse of a transient cavity. The peak pressures developed at collapse can only be calculated from the (force, time) measurements if the area over which the pressure acts is known. At the present time it would seem invidious to draw any conclusions regarding the collapse pressures from the data presented in this report. It is hoped that such information will be forthcoming on completion of phase (ii) (see page 15) of this investigation.

ACKNOWLEDGMENTS

It is a pleasure to express sincere thanks to Dr. A. T. Ellis for many helpful discussions and constant encouragement during the course of this study. The author is also indebted to Professor V. A. Vanoni for his support of this research and to Professor M. S. Plesset for the loan of the high speed camera. Finally, the cooperation and help of the staff of the Hydrodynamics Laboratory is gratefully acknowledged.

REFERENCES

1. Euler, L., Hist, Acad. Roy. Science et Belles Lettres, vol. 10, no. 227, 1754.
2. Reynolds, O., Brit. Ass. Adv. Sci. Report 564; Scientific Papers, vol. 2, no. 578, 1894.
3. Rayleigh, Lord, Phil. Mag., vol. 34, no. 94, 1917.
4. Gilmore, F. R., Hydro. Lab. Report 26-4, Calif. Inst. of Tech., 1952.
5. Parsons, C. A., Trans. Instn. Nav. Arch., vol. 61, no. 223, 1919.
6. Parsons, C. A. and Cook, S. S., Engineering, vol. 107, nos. 501, 515, 1919.
7. Föttinger, H., Hydraulische Probleme, Berlin, Section 27, 1926.
8. Schrotter, H., Z. Ver. Deut. Ing, vol. 76, no. 511, 1932.
9. Schrotter, H., Z. Ver. Deut. Ing, vol. 77, no. 865, 1933.
10. Schrotter, H., Z. Ver. Deut. Ing, vol. 78, no. 349, 1934.
11. de Haller, P., Schweiz. Bauzeit, vol. 101, no. 243, 1933.
12. Mousson, J. M., Trans. A.S.M.E., vol. 59, no. 399, 1937.
13. Gaines, N., Physics, vol. 3, no. 209, 1932.
14. Hunsaker, J. C., Trans. A.S.M.E., vol. 57, no. 423, 1935.
15. Kerr, S. L., Trans. A.S.M.E., vol. 59, no. 373, 1937.
16. Schumb, W. C. et al, Metals and Alloys, vol. 8, no. 126, 1937.
17. Beeching, R., Trans. Instn. Eng. Shipb. Scot., vol. 85, no. 210, 1942.
18. Nowotny, H., Z. Ver. Deut. Ing, vol. 86, no. 279, 1942.
19. Poulter, T. C., J. Appl. Mech., vol. 9, 1942.
20. Cook, S. S., Proc. Roy. Soc. A., vol. 119, no. 481, 1928.
21. Schwarz, M. and Mantel, W., Z. Ver. Deut. Ing, vol. 80, no. 863, 1936.

22. Ackeret, J. and de Haller, P., Schweiz. Bauzeit, vol. 108, no. 105, 1936.
23. Kornfeld, M. and Suvorov, L., J. Appl. Phys., vol. 15, no. 495, 1944.
24. Osborne, M. F. M., Trans. A.S.M.E., vol. 69, no. 253, 1947.
25. Knapp, R. T. and Hollander, A., Trans. A.S.M.E., vol. 70, no. 419, 1948.
26. Meuller, H. F., Kinotechnik, vol. 10, no. 462, 1928.
27. Plesset, M. S., J. Appl. Mech., vol. 16, no. 277, 1949.
28. Chesterman, W. D., Proc. Phys. Soc. B, vol. 65, no. 846, 1952.
29. Harrison, M., J. Acous. Soc. Amer., vol. 24, no. 776, 1952.
30. Mellen, R. H., J. Acous Soc. Amer, vol. 28, no. 447, 1956.
31. Glikman, L. A. et al., Zn. Tekhn, Fiziki, vol. 25, no. 280, 1955.
32. Plesset, M. S. and Ellis, A. T., Trans. A.S.M.E., vol. 77, no. 1055, 1955.
33. Ellis, A. T., Proc. Symp. "Cavitation in Hydrodynamics", N.P.L. London: H.M.S.O. Paper No. 8, 1956.
34. Sutton, G. W., J. Appl. Mech., vol. 24, no. 340, 1957.
35. Jones, I. R. and Edwards, D. H., J. Fluid Mech., vol. 7, no. 596, 1960.
36. Güth, W., Acustica, vol. 6, no. 526, 1956.
37. Eisenberg, P., Proc. Symp. "Cavitation in Hydrodynamics", N.P.L. London: H.M.S.O. Paper 1, 1956.
38. Hopkinson, B., Phil. Trans. A, vol. 213, no. 437, 1914.
39. Davies, R. M., Phil. Trans. A, vol. 240, no. 375, 1948.
40. Thomas, D. E., Ph.D. Thesis (Univ. of Wales), 1950.
41. Ripperger, E. A., Tech. Report No. 13, Div. of Eng. Mechs., Univ. of Stanford, 1952.
42. Edwards, D. H., J. Sci. Inst., vol. 35, no. 346, 1958.
43. Mason, W. P., Piezoelectric crystals and their application to ultrasonics, p. 85 (New York: van Nostrand Co.), 1950.

44. Ellis, A. T., Private Communication, 1959.

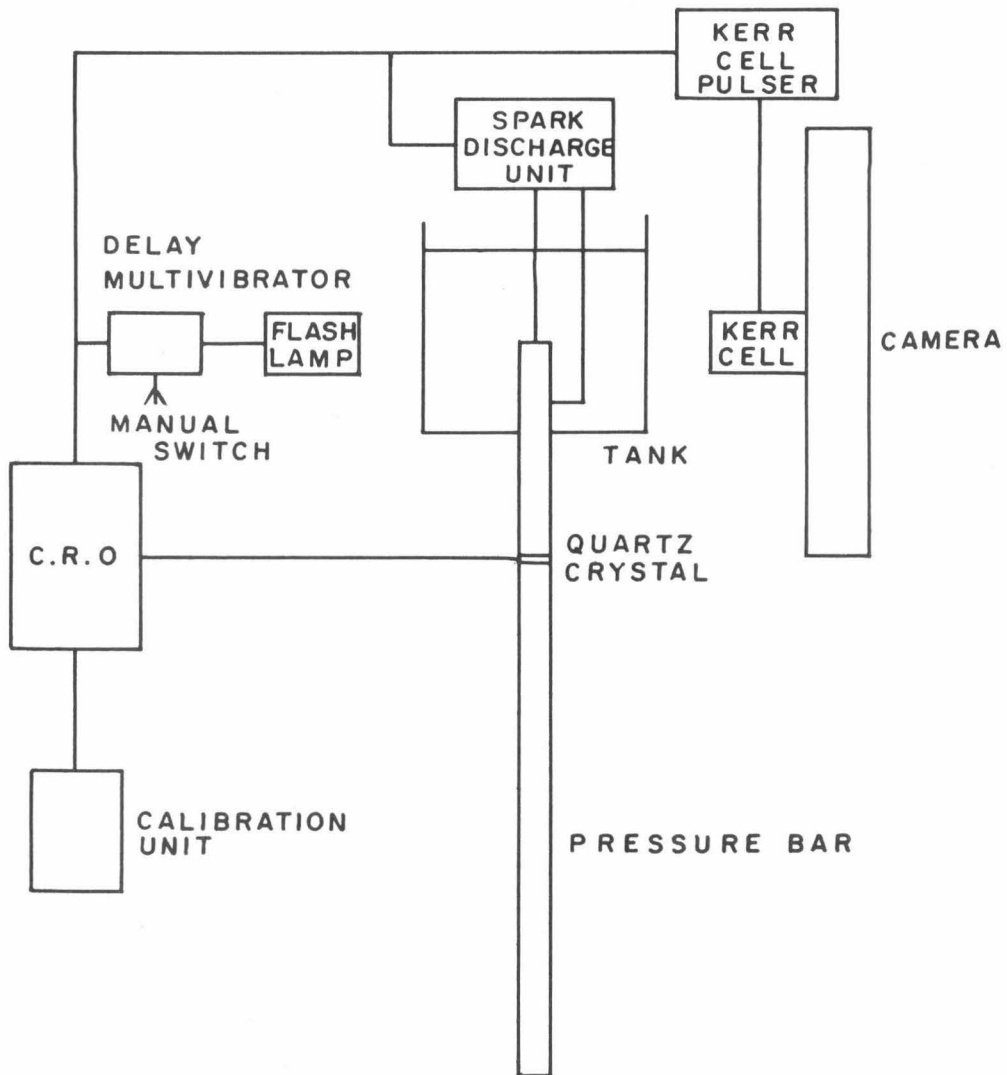


Fig. 1. Schematic diagram of apparatus.

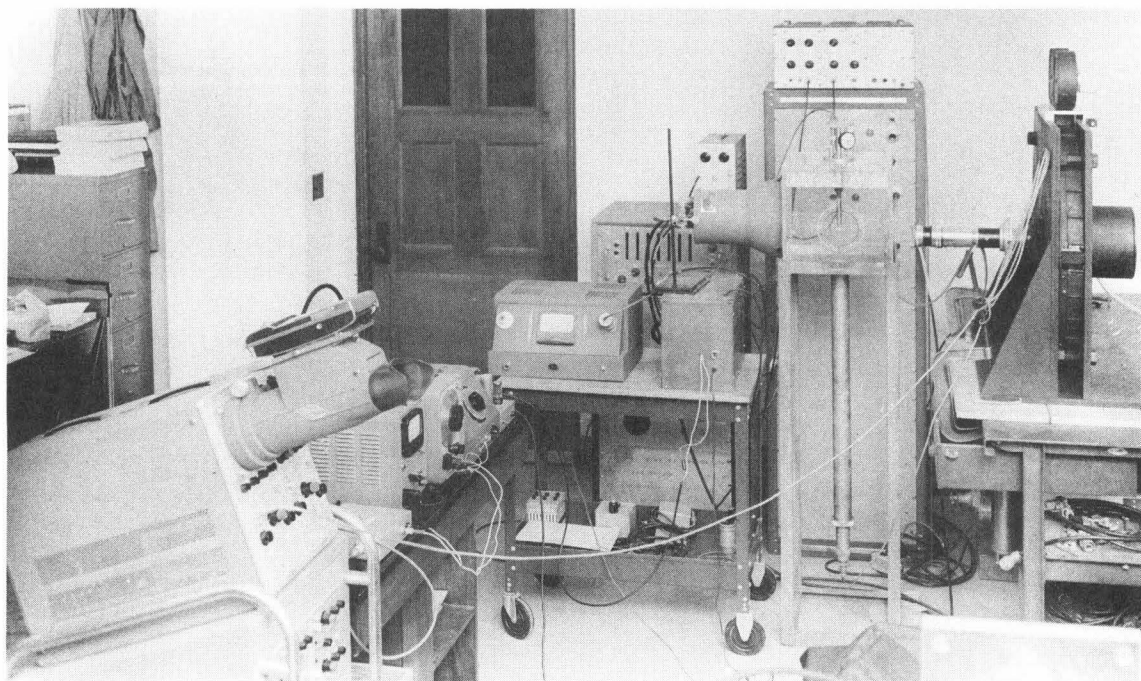


Fig. 2. Experimental apparatus.

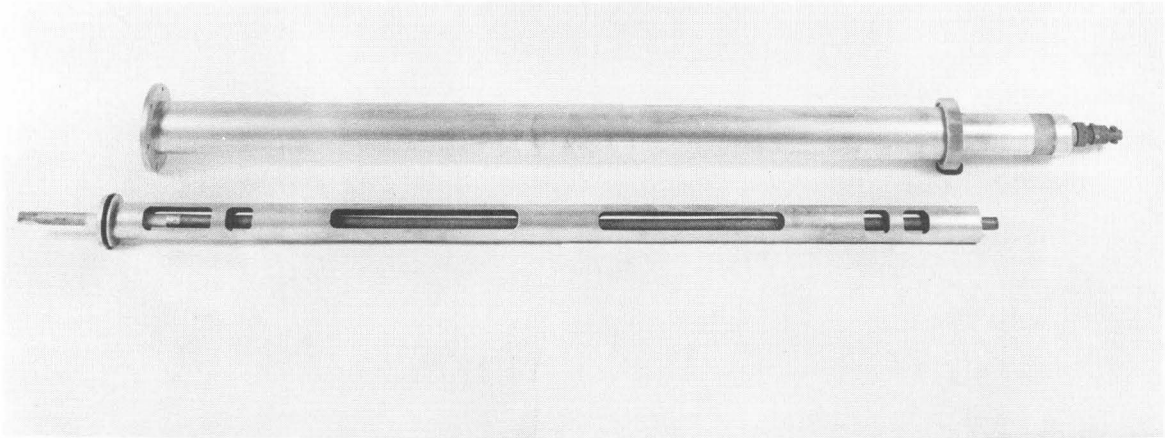


Fig. 3. 1/2-inch pressure bar and associated housing.

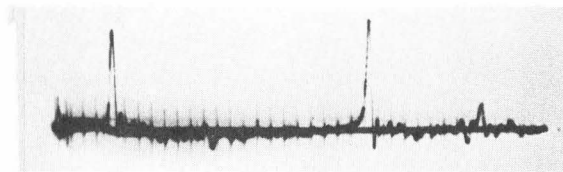
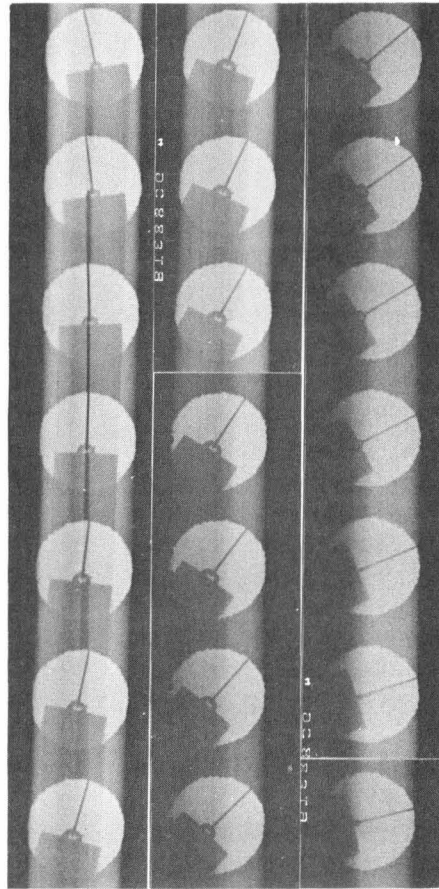


Fig. 4. Typical experimental data obtained using the 1/4-inch diameter pressure bar. Cavity lifetime, $T : 198 \text{ } \mu\text{s}$; peak force, F_m , developed at the collapse of the cavity : 2.63×10^6 dynes; maximum radius, R_0 , attained by cavity : 0.112 cms; Camera framing rate : 100,000 frames/sec; frequency of timing markers on oscillogram : 100 Kc/s.

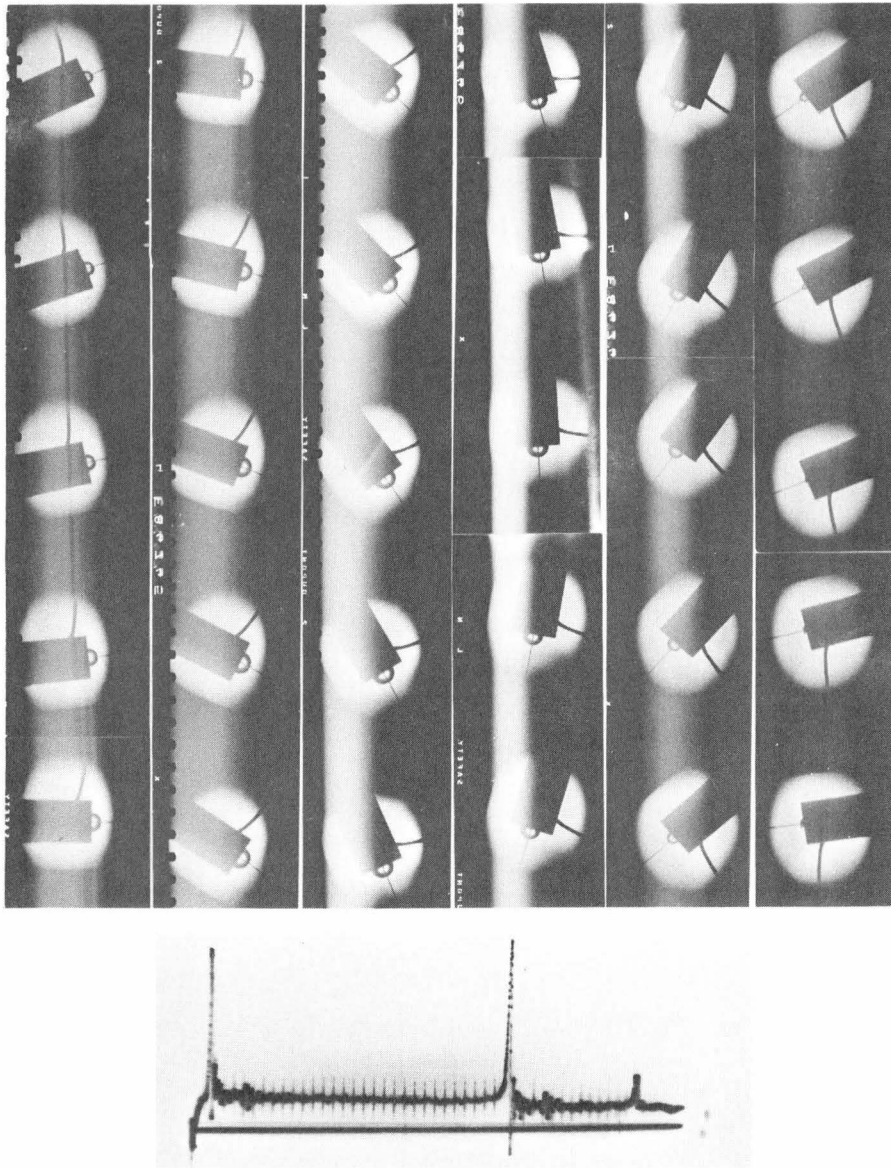


Fig. 5. Typical experimental data obtained using the 1/2-inch diameter pressure bar. Cavity lifetime, T : 603 μ s; peak force, F_m , developed at the collapse of the cavity : 33.9×10^6 dynes; maximum radius, R_0 , attained by cavity : 0.338 cms; camera framing rate : 50,000 frames/sec; frequency of timing markers on oscillogram : 50 Kc/s.

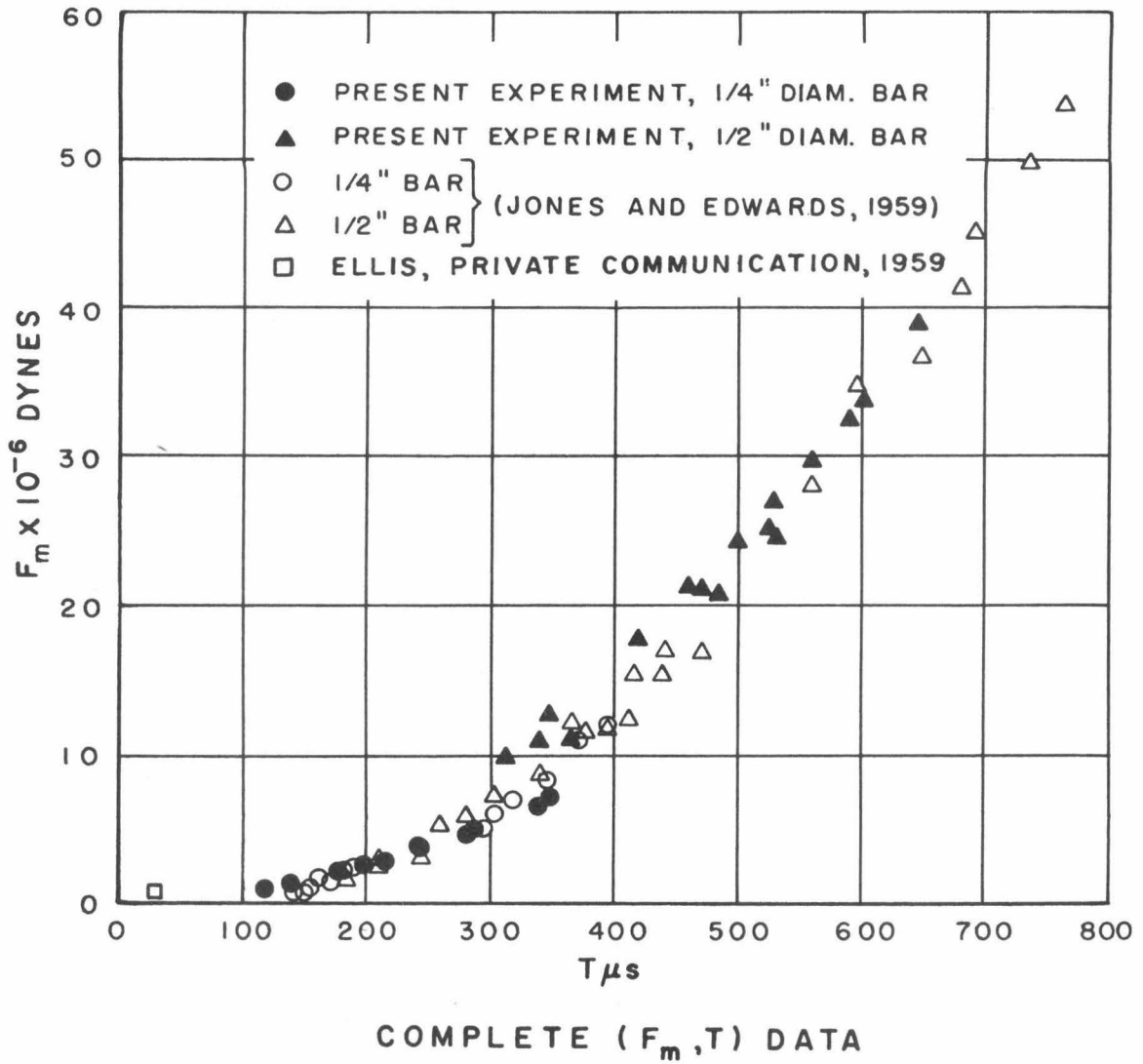


Fig. 6.

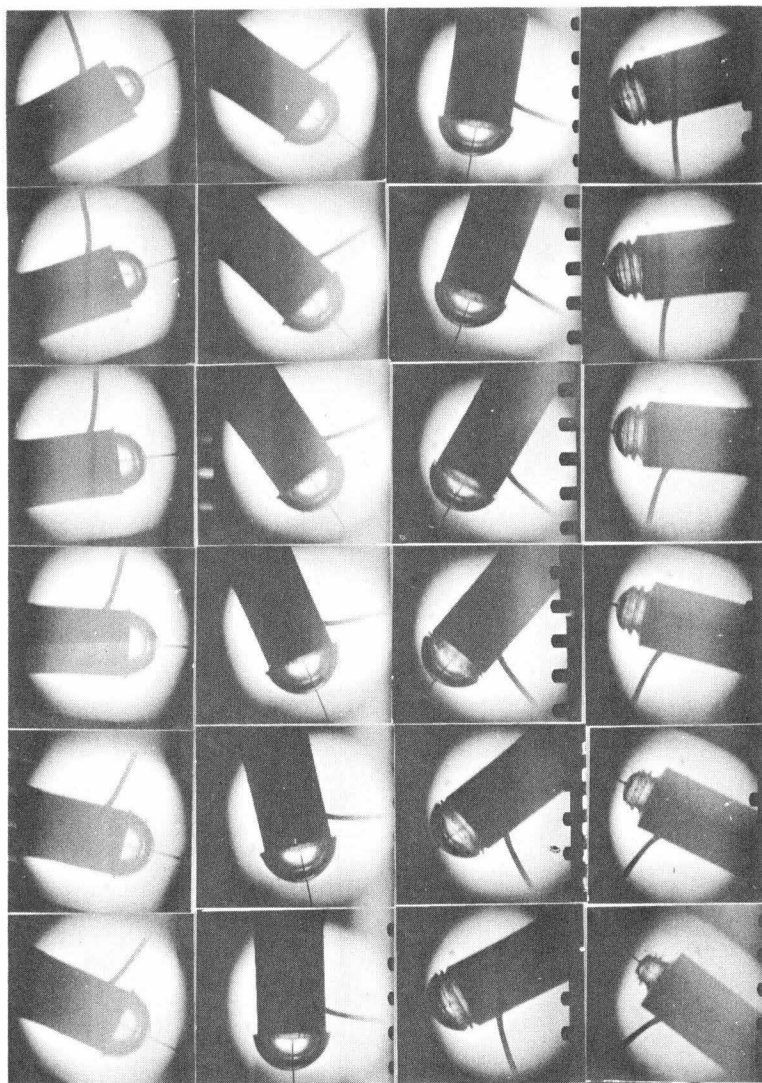


Fig. 7. The growth and collapse of a cavity which attained a diameter larger than the diameter of the pressure bar. Diameter of pressure bar : $1/2$ inch; camera framing rate : 50,000 frames/sec.

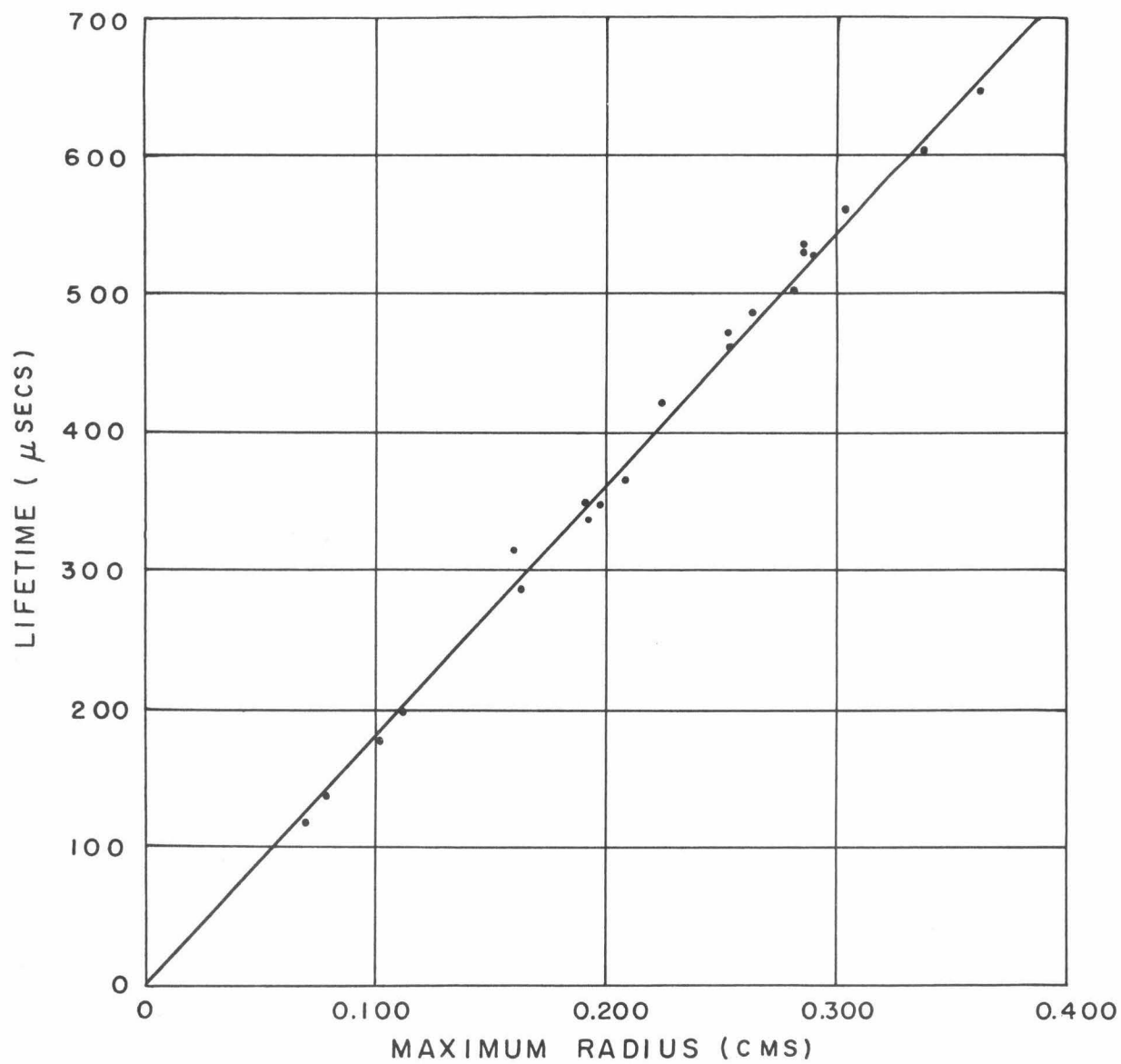


Fig. 8.

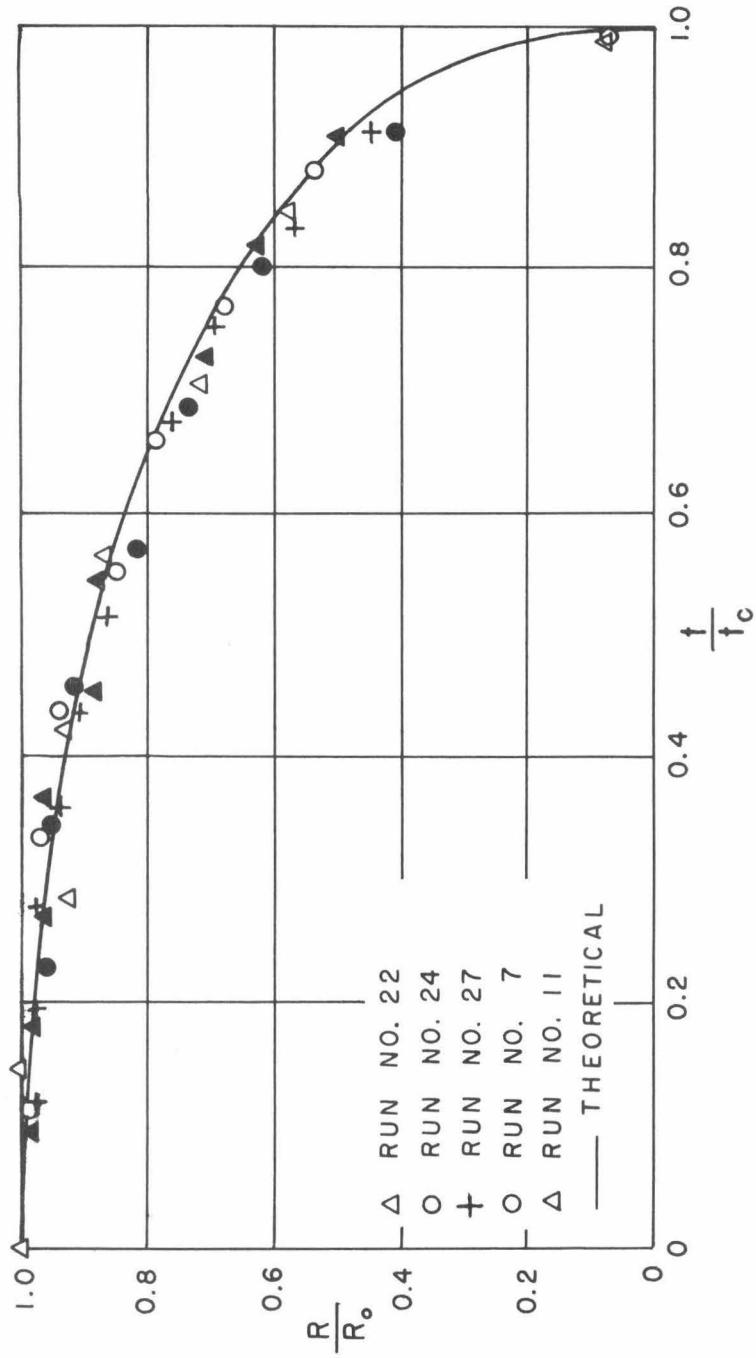


Fig. 9.

Table 1. SUMMARY OF RESULTS

Run No.	Diameter of bar used in run in.	Capacity of condenser μf	Charging potential KV	Energy stored in condenser joules	Maximum radius R_0 attained by cavity cms	Lifetime T of cavity μs	Peak force F^m developed at collapse of cavity $\times 10^{-6}$ dynes
1	0.5	0.02	2.0	0.04	0.191	348	12.7
2	0.5	0.02	4.0	0.16	0.254	460	21.3
3	0.5	0.02	6.0	0.36	0.338	603	33.9
4	0.5	0.02	8.0	0.64	0.305	560	29.7
5	0.5	0.02	10.0	1.00	0.366	647	39.0
6	0.5	0.02	2.0	0.04	0.208	365	11.0
7	0.5	0.02	2.0	0.04	0.188	340	10.9
8	0.5	0.02	2.0	0.04	0.160	313	9.0
9	0.5	0.02	4.0	0.16	0.254	471	21.2
10	0.5	0.02	6.0	0.36	0.287	530	27.0
11	0.5	0.02	3.0	0.09	0.224	420	17.8
12	0.5	0.02	5.0	0.25	0.282	501	24.2
13	0.5	0.035	5.0	0.44	0.328	592	32.5
14	0.5	0.035	4.0	0.28	0.290	526	25.3
16	0.5	0.035	4.0	0.28	0.287	533	24.5
17	0.5	0.035	3.0	0.16	0.264	485	20.8
22	0.25	0.005	5.0	0.06	0.079	138	1.39
23	0.25	0.005	3.0	0.02	0.069	118	1.02
24	0.25	0.005	5.0	0.06	0.102	176	2.13
25	0.25	0.005	7.0	0.12	0.119	215	2.87
26	0.25	0.005	7.0	0.12	0.112	198	2.63
27	0.25	0.005	10.0	0.25	0.135	244	3.72
29	0.25	0.007	7.0	0.17	0.163	287	5.04
30	0.25	0.007	7.0	0.17	0.193	337	6.48
32	0.25	0.007	7.0	0.17	0.198	348	7.12
34	0.25	0.007	7.0	0.17	--	281	4.62

Table 2. RATIO OF GROWTH TO COLLAPSE TIMES

Run No.	Maximum radius R_0 (cms)	T (μ s)	t_g (μ s)	t_c (μ s)	t_g/t_c
22	0.079	138	67	71	0.95
24	0.102	176	89	91	0.94
27	0.135	244	119	125	0.95
7	0.188	340	165	175	0.94
11	0.224	420	200	220	0.91

Mean value of $t_g/t_c = 0.94$.